Specificity and context dependent preferences in argumentation systems

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Abstract. Dung and Son [6] argue that *specificity* as a criterion for resolving conflicts between arguments, is context dependent. They propose to use arguments to address the context dependency of specificity in combination with a new special argumentation semantics. Unfortunately, their solution is restricted to argumentation systems without undercutting arguments. This paper presents a more general solution which allows for undercutting arguments and allows for any argumentation semantics. Moreover, the solution is applicable to any form a context dependent preferences.

keywords: specificity, argumentation, context dependent preferences

1 Introduction

Specificity Dung and Son [6] argue that *specificity* as a criterion for resolving conflicts between arguments, is context dependent. They illustrate their point with the following example:

1. Students are normally not married.
2. Students are normally young adults.
3. Young adults are adults.
4. Adults are normally married.

The first and the fourth sentence support contradictory conclusions. Given that someone is a student, we can construct an argument for *being an adult*. Therefore, *being a student* is more specific than *being an adult*, implying that the first sentence is preferred to the fourth sentence [6, 10, 17, 18]. However, knowing that someone *is a student and an adult but not a young adult*, the specificity preference should not be valid. The solution that Dung and Son [6] propose, is:

1. to represent an argument by the defeasible sentences (the defeasible rules) used to derive a conclusion,
2. to let an argument attack a defeasible rule instead of other arguments,

3. to register the argument for a rule to be more specific than another rule (the argument for the antecedent of the more general rule given the antecedent of the more specific rule), and
4. to attack the argument for the conclusion of the more general rule based on specificity.

If a student is an adult but not a young adult, the second sentence is not applicable (is attacked and defeated) and therefore the argument that the first sentence is more specific than the fourth sentence, is not valid. Note that the first and fourth sentence attack each other (rebuttal) and that the attack on the fourth sentence based on specificity, ensures that the more general sentence (rule) is defeated if the specificity argument is valid.

A problem with the solution of Dung and Son [6] is that it fails to handle undercutting attacks correctly. Suppose that the first sentence is not applicable for students from the country Utopia, and that we model this with an undercutting attack. Given a student from Utopia, the first sentence in not applicable, but because the first sentence is more specific than the fourth sentence, indirectly, it still successfully attacks the fourth sentence. This is not what we expect. The cause of the problem is that an attack based on specificity only considers whether the situation in the first sentence (the antecedent of the first rule) is more specific than the situation in the fourth sentence (the antecedent of the fourth rule). It does not consider whether the more specific sentence (rule) is applicable. Without undercutting attacks, this is not a problem because any attacking argument will support the opposite conclusion. Consider for instance the sentence: "Students from Utopia are normally married".

To address the above outlined problem, we will investigate the following solution:

- We use an argumentation system in which undercutting arguments attack the application of a defeasible rule. Such arguments attack every argument in which the attacked defeasible rules is used [17, 18]. This approach is in line with the solution of Dung and Son [6] where a set of defeasible rules represents the argument that attacks a defeasible rule. An advantage of this argumentation system is the existence of a semantic tableau method for generating arguments [20].
- We explicitly *assume the absence* of a preference when we construct undercutting arguments that resolve derived conflicts (inconsistencies). In case of the above example, we assume: “Students are normally not married” **is not preferred to** “Adults are normally married”, and assume: “Adults are normally married” **is not preferred to** “Students are normally not married”. In the absence of preferences, multiple undercutting arguments may be formulated, which may result in multiple extension [17, 18].
- We use explicit arguments for (specificity based) preferences. For instance an argument for: “Students are normally not married” **is preferred to** “Adults are normally married”. These arguments may attack the assumptions mentioned in the previous item.

Context dependent preferences Specificity is a special case of context dependent preferences. Since a specificity preference is essentially not different from other context dependent preferences, the above outlined approach may also be used for general context dependent preferences. It will offer an alternate for several approaches described in the literature [7, 11, 14]. These approaches have in common that they use special procedures for handling the derived preferences.

– Prakken and Sartor [14] focus on the grounded semantics. They make use of the property that the least fixed point of the characteristic function can be computed by repeatedly applying the characteristic function, starting from the empty set, to the result from the previous application. Each application monotonically extends the set of 'justified arguments'. The justified preferences after one iteration are used by the characteristic function to determine justified arguments of the next iteration.
– Modgil [11] proposes a more general approach. He extends Dung's is argumentation framework order to incorporate defensible preferences over arguments. The preferences are used to introduce a defeat relation, which is an attack relation that is consistent with preferences supported by a set of arguments. This definition has been criticized by Amgoud and Vesic [1], who propose a modified definition that solves the problem they identified. Beside introducing a defeat relation, Modgil adapts Dung's definition [5] of an argument that is acceptable w.r.t. a set of arguments. Other definitions of Dung's argumentation semantics are not changed. Modgil's approach leaves open how preferences over defeasible rules are mapped to preferences over arguments.
– Dung et al. [7] present a different approach to handling defeasible preferences. They allow an argument for a preference between two rules, to attack an attack relation between two arguments that use the two rules, which they call *preference attacks*. This approach makes it possible to use a standard argumentation framework [5] given a set of preference arguments that we accept. To select the set of preference arguments that we accept, Dung et al. [7] formulate postulates for preference attacks and select the smallest set of arguments supporting preferences satisfying these postulates.

The main difference between the approach investigated in this paper and several other approaches is in the view of an argument. Following Dung [5], an argument is often viewed as an atom. Although this offers important advantages, one should keep in mind that an argument is defeasible because it is build using defeasible element, usually, defeasible rules [17, 18]. Preferences are not about arguments but about these defeasible elements. Therefore, an approach that focuses on these defeasible elements may have benefits over other approaches.

Outline The next section describes the preliminaries. It introduces the arguments and the argumentation system used in the paper. A semantic tableau method for generating arguments of the argumentation system has been described in [20]. Section 3 describes the handling of context dependent (specificity) preferences and Section 4 evaluates the proposed approach. Section 5 concludes the paper.

2 Preliminaries

This section presents the argumentation system that will be used in the discussion of a general solution for context dependent preferences, including specificity.

We assume a standard logic such as propositional or predicate logic. The language of the logic will be denoted by $\mathcal{L}$. We also assume that the language $\mathcal{L}$ contains the symbols $\top$ denoting *true*, and $\bot$ denoting *false*. In case of predicate logic, the set of ground terms is denoted by $\mathcal{G}$.

Since this paper focuses on using context dependent preferences for resolving conflicts between arguments, we need a definition of an argument. Toulmin [22] views an argument as a support for some *claim*. The support is grounded in *data*, and the relation between the data and the claim is the *warrant*. Here, we use the following definition.

Definition 1. *A couple $A = (\mathcal{S}, \varphi)$ is called an argument where φ is said to be its conclusion, and $\mathcal{S}$ is a set said to be its support; its elements are called supporting elements. It is worthwhile observing here that this definition is very general and a many couples might be qualified as arguments.*

In case of propositional and predicate logic, the support $\mathcal{S}$ is a set of propositions from the language $\mathcal{L}$. Generally, $\mathcal{S}$ contains the set of premises used to derive the supported proposition φ. So, $\mathcal{S} \vdash \varphi$. In special applications, such as Model-Based Diagnosis, we may restrict $\mathcal{S}$ to assumptions about the normal behavior of components.

We may extend a standard logic with a set of defeasible rules. Defeasible rules are of the form:

$$\varphi \leadsto \psi$$

in case of propositional logic, and of the form:

$$\varphi(\mathbf{x}) \leadsto \psi(\mathbf{x})$$

in case of predicate logic. Here, φ is propositions from the language $\mathcal{L}$, ψ is either a proposition from the language $\mathcal{L}$ or a negated defeasible rule of the form: $\mathbf{not}(\eta \leadsto \mu)$, and $\mathbf{x}$ is a sequence of free variables. The free variables denote a set of ground instances of the defeasible rule $\varphi(\mathbf{x}) \leadsto \psi(\mathbf{x})$. We do not use the universal quantifier because the rule is not a proposition that belongs to the language $\mathcal{L}$. It is an additional statement that need not be valid for every ground instance.

The defeasible rules $\varphi \leadsto \mathbf{not}(\eta \leadsto \mu)$ and $\varphi(\mathbf{x}) \leadsto \mathbf{not}(\eta(\mathbf{x}) \leadsto \mu(\mathbf{x}))$ are called *undercutting defeaters* [12]. These undercutting defeaters specify the conditions φ and $\varphi(\mathbf{x})$ under which the defeasible rules $\eta \leadsto \mu$ and $\eta(\mathbf{x}) \leadsto \mu(\mathbf{x})$ respectively, are not applicable.

We use $\Sigma \subseteq \mathcal{L}$ to denote the set of available information and we use D to denote the set of available rules. Moreover, we use $\overline{D} = \{\varphi(\mathbf{t}) \leadsto \psi(\mathbf{t}) \mid \varphi(\mathbf{x}) \leadsto \psi(\mathbf{x}) \in D, \mathbf{t} \in \mathcal{G}^n\}$ to denote the set of ground instances of the defeasible rules with n free variables in case of predicate logic, and $\overline{D} = D$ in case of propositional logic.

Defeasible rules are used in the construction of arguments. Whenever we have a support $\mathcal{S}'$ for the antecedent φ of a defeasible rule $\varphi \rightsquigarrow \psi$, we can create a supporting element $(\mathcal{S}', \varphi \rightsquigarrow \psi)$, which can be used to support ψ. The arguments that can be constructed are defined as:

Definition 2. *Let $\Sigma \subseteq \mathcal{L}$ be the initial information and let D be a set of defeasible rules. An argument $A = (\mathcal{S}, \psi)$ with premises $\bar{A}$, defeasible rules $\tilde{A}$, last defeasible rules $\vec{A}$, supported proposition (claim / conclusion) $\hat{A}$, and supporting propositions $\hat{\mathcal{S}}$ of $\hat{A}$, is recursively defined as:*

- *If $\psi \in \Sigma$, then $A = (\{\psi\}, \psi)$ is an argument.*
  $\bar{A} = \{\psi\}$. $\tilde{A} = \varnothing$. $\hat{A} = \psi$. $\hat{\mathcal{S}} = \{\psi\}$.
- *If $A_1 = (\mathcal{S}_1, \varphi_1), \ldots, A_k = (\mathcal{S}_k, \varphi_k)$ are arguments and $\{\varphi_1, \ldots, \varphi_k\} \vdash \psi$, then $A = (\mathcal{S}_1 \cup \cdots \cup \mathcal{S}_k, \psi)$.*
  $\bar{A} = \bar{A}_1 \cup \cdots \cup \bar{A}_k$. $\tilde{A} = \tilde{A}_1 \cup \cdots \cup \tilde{A}_k$. $\vec{A} = \vec{A}_1 \cup \cdots \cup \vec{A}_k$. $\hat{A} = \psi$. $\hat{\mathcal{S}} = \hat{\mathcal{S}}_1 \cup \cdots \cup \hat{\mathcal{S}}_k$.
- *If $A' = (\mathcal{S}', \varphi)$ is an argument and $\varphi \rightsquigarrow \psi \in \overline{D}$ is a defeasible rule, then $A = (\{(\mathcal{S}', \varphi \rightsquigarrow \psi)\}, \psi)$ is an argument.*
  $\bar{A} = \bar{A}'$. $\tilde{A} = \{\varphi \rightsquigarrow \psi\} \cup \tilde{A}'$. $\vec{A} = \{\varphi \rightsquigarrow \psi\}$. $\hat{A} = \psi$. $\hat{\mathcal{S}} = \{\psi\}$.

$A = (\mathcal{S}, \psi)$ is a minimal *argument iff (1) $\mathcal{S}$ is a minimal set such that $\hat{\mathcal{S}} \vdash \psi$, and (2) for every $(\mathcal{S}', \alpha \rightsquigarrow \beta) \in \mathcal{S}$, $(\mathcal{S}', \alpha)$ is a minimal argument.*

Note that for every argument, there exists a corresponding minimal argument supporting the same conclusion.

This abstract representation of arguments is based on the representation of arguments proposed in [17, 18]. It assumes that the derivation relation $\vdash$ of the underlying logic is sound and complete. This ensures that inconsistencies do not remain hidden because of the chosen formulation. A reasoning process based call an argumentation tableau, which is based on the construction of a semantic tableau, has been proposed for this argumentation system [20].

We will use a graphical representation of an argument for human readability. The argument for an inconsistency:

$$\begin{aligned} A = \ & (\{(\{(\{(\{p \vee q, \neg q\}, p \rightsquigarrow r), (\{s\}, s \rightsquigarrow t)\}, r \wedge t \rightsquigarrow u), \\ & (\{v\}, v \rightsquigarrow w), \neg(u \wedge w)\}, \bot) \end{aligned}$$

is graphically represented as:

$$A = \left[\begin{array}{r} \left.\begin{array}{r} \left.\begin{array}{r} p \vee q \\ \neg q \end{array}\right| \vdash p \rightsquigarrow r \\ \\ s \vdash s \rightsquigarrow t \end{array}\right| \vdash r \wedge t \rightsquigarrow u \\ \\ v \vdash v \rightsquigarrow w \\ \neg(u \wedge w) \end{array}\right| \vdash \bot \ \right]$$

Here, $\hat{A} = \bot$, $\vec{A} = \{r \wedge t \rightsquigarrow u, v \rightsquigarrow w\}$, $\tilde{A} = \{p \rightsquigarrow r, s \rightsquigarrow t, r \wedge t \rightsquigarrow u, v \rightsquigarrow w\}$, $\bar{A} = \{p \vee q, \neg q, s, v, \neg(u \wedge w)\}$ and $\hat{\mathcal{S}} = \{u, w, \neg(u \wedge w)\}$ with $A = (\mathcal{S}, \bot)$. Note that we use $\vdash$ in the graphical representation to denote standard deduction. We will use $\mid\!\approx$ instead of $\vdash$ for derivations that are neither deductive nor the result of applying a defeasible rule, as in Definitions 3, 4 and 5.

When an argument for an inconsistency is derived[1], one of the defeasible rules is not applicable in the current context. If no defeasible rule is involved in the argument for the inconsistency, one of the premises is invalid. In both cases we will use a strict partial order $<$ on the defeasible rules D and on the information in Σ to determine the rule and premise that is invalid, respectively. Note that context dependent preferences will be added in subsequent sections and $<$ can be an empty set of preferences. Following [15–18], we formulate an *undercutting* argument for the culprit. That is, an argument attacking every argument that uses the culprit.[2]

Definition 3. *Let $A = (\mathcal{S}, \perp)$ be an argument for an inconsistency. Moreover, let $< \subseteq (\Sigma \times \Sigma) \cup (D \times D)$ be a strict partial order over the information Σ and over the defeasible rules D. Finally, let $A' = (\mathcal{S}', \mathbf{not}(\varphi \rightsquigarrow \psi))$ and $A' = (\mathcal{S}', \mathbf{not}(\sigma))$ denote the arguments for an undercutting attack of a defeasible rule in $\overline{D}$ and a proposition in Σ respectively.*

- *If $\tilde{A} \neq \varnothing$, **defeat the weakest last rule**. For every $\varphi \rightsquigarrow \psi \in min_<(\vec{A})$ with $(\mathcal{S}'', \varphi \rightsquigarrow \psi) \in \mathcal{S}$, $A' = (\mathcal{S} \backslash (\mathcal{S}'', \varphi \rightsquigarrow \psi), \mathbf{not}(\varphi \rightsquigarrow \psi))$ is an undercutting argument of $\varphi \rightsquigarrow \psi \in \overline{D}$.*
- *If $\tilde{A} = \varnothing$, **defeat the weakest premise**. For every $\sigma \in min_<(\bar{A})$, $A' = (\mathcal{S} \backslash \sigma, \mathbf{not}(\sigma))$ is an undercutting argument of $\sigma \in \Sigma$.*

Note that $min_<(\cdot)$ need not be unique because $<$ is a strict partial order. Also note that $\mathcal{S} \backslash (\mathcal{S}', \varphi \rightsquigarrow \psi)$ is an argument for $\neg\psi$, and that $\mathcal{S} \backslash \sigma$ is an argument for $\neg\sigma$.

The undercutting arguments define an attack relation over the arguments. We denote the attack relation over a set of arguments $\mathcal{A}$ by $\longrightarrow \subseteq \mathcal{A} \times \mathcal{A}$. An undercutting argument $A = (\mathcal{S}, \mathbf{not}(\varphi \rightsquigarrow \psi))$ attacks every argument A' for which $\varphi \rightsquigarrow \psi \in \tilde{A}'$ holds. Moreover, an undercutting argument $A = (\mathcal{S}, \mathbf{not}(\sigma))$ attacks every argument A' for which $\sigma \in \bar{A}'$ holds. We denote the attack of A on A' by $A \longrightarrow A'$. The set of all derived arguments $\mathcal{A}$ and the attack relation over the arguments $\longrightarrow \subseteq \mathcal{A} \times \mathcal{A}$ determine an instance of an argumentation framework $(\mathcal{A}, \longrightarrow)$ as defined by Dung [5]. We can use one the semantics for argumentation frameworks to determine sets of valid arguments; i.e., the argument extensions. See for instance: [2–5, 8, 9, 19, 23].

3 Context dependent preferences

We first address *specificity*, which is a specific form of context dependent preferences. Next we discuss general context dependent preferences. We conclude

[1] Arguments for inconsistencies cover rebutting attacks.

[2] Note the difference between an undercutting argument and an undercutting defeater. The former is an argument for not using a proposition or a defeasible rule, and the latter is a defeasible rule specifying a condition under which another defeasible rule should not be used [12].

with a description of the changes to the argumentation system described in the previous section.

3.1 Specificity

Specificity is the principle by which rules applying to situations that are more specific, override those applying to situations that are more general. In other words, what holds in a specific situation may represent an exception on what holds in the more general situation. To determine whether we have a specificity preference between two defeasible rules, we must determine whether the situation in which one rule is applicable implies the situation in which the other rule is applicable. For this we may use the general knowledge described by the defeasible rules D and by the background knowledge $\mathcal{K} \subseteq \mathcal{L}$. Therefore, to determine whether a rule $\varphi \rightsquigarrow \psi$ is preferred to a rule $\eta \rightsquigarrow \mu$ based on a specificity preference, we have to check whether we can construct an argument for the antecedent η given the antecedent φ, using the defeasible rules D and by the background knowledge $\mathcal{K}$ [6, 10, 13, 18, 21].

Suppose that we have the defeasible rules $D = \{\varphi \rightsquigarrow \psi, \eta \rightsquigarrow \mu, \varphi \rightsquigarrow \eta\}$ with $\{\psi, \mu\} \cup \mathcal{K} \vdash \bot$. Then, assuming φ, we can construct an argument for η:

$$A^{\eta} = \left[\varphi \vdash \varphi \rightsquigarrow \eta \vdash \eta\right]$$

This implies that the situation described by φ is more specific that the situation described by η, and therefore, $\varphi \rightsquigarrow \psi$ must be preferred to $\eta \rightsquigarrow \mu$ because of specificity [6, 10, 13, 17, 18, 21]. Of course, we must make sure that the situation described by φ is *strictly* more specific that the situation described by η. Therefore, given η, we should not be able to derive an argument for φ.

There are two aspects that we need to consider using this approach. First, for no sub-argument $A' = (\mathcal{S}', \alpha)$ of A^{η}, $\hat{\mathcal{S}}'$ may be inconsistency [6]. If $\hat{\mathcal{S}}'$ is inconsistent, a rule in $\vec{A}'$, and therefore a rule in $\tilde{A}^{\eta}$ must be defeated. To give an illustration, the following specificity argument for η is *not* allowed:

$$A^{\eta} = \left[\begin{array}{r|l} \varphi \vdash \varphi \rightsquigarrow \alpha \vdash \alpha \rightsquigarrow \neg\beta & \\ \varphi \vdash \varphi \rightsquigarrow \beta & \vdash \eta \end{array} \right]$$

Second, the specificity argument A^{η} supporting that $\varphi \rightsquigarrow \psi$ is preferred to $\eta \rightsquigarrow \mu$ may not be defeated by another argument [6]. Repeating the example mention in the Introduction, suppose that we have the defeasible rules: students are normally not married, students are normally young adults, and adults are normally married.

$$\begin{array}{l} \textit{student} \rightsquigarrow \neg\textit{married} \\ \textit{student} \rightsquigarrow \textit{young adult} \\ \textit{adult} \rightsquigarrow \textit{married} \end{array}$$

The first and the last rule support conflicting conclusions. Since being a student is more specific than being a young adult, which is more specific than being an adult, the first rule should be preferred to the last rule.

$$A = \left[\textit{student} \vdash \textit{student} \rightsquigarrow \textit{young adult} \vdash \textit{adult}\right]$$

However, if we know that someone is a student and an adult but not a young adult, then this specificity preference is no longer valid for this student.

The following definition specifies the argument for the specificity preference.

Definition 4. *Let D be a set of defeasible rules, let $\mathcal{K} \subseteq \Sigma$ be the background knowledge, let $\varphi \rightsquigarrow \psi, \eta \rightsquigarrow \mu$ be two rules in D, and let $A^\varphi = (\mathcal{S}^\varphi, \varphi)$ be an argument for φ.*

$A = (\mathcal{S}, \eta \rightsquigarrow \mu < \varphi \rightsquigarrow \psi)$ is an argument for preferring $\varphi \rightsquigarrow \psi$ to $\eta \rightsquigarrow \mu$ based on specificity *if and only if*

- *given the information $\{\varphi\} \cup \mathcal{K}$, there exists an argument $A^\eta = (\mathcal{S}^\eta, \eta)$ (note that $\bar{A}^\eta \subseteq \{\varphi\} \cup \mathcal{K}$),*
- *for **no** sub-argument $A' = (\mathcal{S}', \alpha)$ of A^η, $\hat{\mathcal{S}}' \vdash \bot$,*
- *given the information $\{\eta\} \cup \mathcal{K}$, there **does not** exists an argument $A' = (\mathcal{S},' \varphi)$ (note that $\bar{A}' \subseteq \{\eta\} \cup \mathcal{K}$), and*
- *$\mathcal{S}$ is the result of replacing every occurrence of φ in the support $\mathcal{S}^\eta$ by the support $\mathcal{S}^\varphi$.*

Since φ may not be part of the given information (i.e., $\varphi \notin \Sigma$), the last item in the above definition ensures that we have a proper argument for the specificity preference.

As an illustration, consider the defeasible rules $D = \{\alpha \rightsquigarrow \varphi, \varphi \rightsquigarrow \psi, \varphi \rightsquigarrow \eta, \eta \rightsquigarrow \neg\psi\}$ and the available information $\Sigma = \{\alpha\}$. Given φ, we can derive the argument $A^\eta = \left[\varphi \vdash \varphi \rightsquigarrow \eta \vdash \eta\right]$. Clearly, $\bar{A}^\eta \subseteq \{\varphi\} \cup \mathcal{K}$. Moreover, A^η has no sub-arguments supporting an inconsistency, and we cannot derive an argument A' for φ with $\bar{A}' \subseteq \{\eta\} \cup \mathcal{K}$. Therefore, $\varphi \rightsquigarrow \psi$ is preferred to $\eta \rightsquigarrow \neg\psi$ based on specificity. Using the argument $A^\varphi = [\alpha \vdash \alpha \rightsquigarrow \varphi \vdash \varphi]$, we can construct the following argument for this preference:

$$A = [\alpha \vdash \alpha \rightsquigarrow \varphi \vdash \varphi \rightsquigarrow \eta \not\models \eta \rightsquigarrow \mu < \varphi \rightsquigarrow \psi]$$

There are two practical issue concerning Definition 4 that we need to address. Definition 4 contains a consistency test and a derivability test. Both tests can easily be carried out for proposition logic, for instance using the argumentation tableau [20]. However, since predicate logic is semi-decidable, these tests raise a problem. Fortunately, an argumentation based approach offers a solution. Because in the last item of Definition 4, we construct a proper argument $A = (\mathcal{S}, \eta \rightsquigarrow \mu < \varphi \rightsquigarrow \psi)$ for the specificity preference of which the premises $\bar{A}$ are a subset of the given information Σ, for each sub-argument of A^η supporting an inconsistency, there is a corresponding sub-argument of A supporting an inconsistency. When this sub-argument is derived, Definition 3 is applied ensuring that this inconsistency is avoided, thereby addressing the consistency test.

The derivability test, needed to ensure that the specificity preference is strict, forms a bigger challenge. We can address it by adding to the definition of the preference argument A in Definition 4, the *assumption*, denoted by the keyword ***assume***, that there is no valid preference for the opposite. If we do derive an argument for such a preference, it will attack argument A. Based on the above suggested ways to handle the two issues, we modify Definition 4.

Definition 5 (Definition 4 revised). *Let D be a set of defeasible rules, let $\mathcal{K} \subseteq \Sigma$ be the background knowledge, let $\varphi \rightsquigarrow \psi, \eta \rightsquigarrow \mu$ be two rules in D, and let $A^\varphi = (\mathcal{S}^\varphi, \varphi)$ be an argument for φ.*

$$A = (\mathcal{S} \cup \{\boldsymbol{assume}(\varphi \rightsquigarrow \psi \not< \eta \rightsquigarrow \mu)\}, \eta \rightsquigarrow \mu < \varphi \rightsquigarrow \psi)$$

is an argument for preferring $\varphi \rightsquigarrow \psi$ to $\eta \rightsquigarrow \mu$ based on specificity *if and only if*

- *given the information $\{\varphi\} \cup \mathcal{K}$, there exists an argument $A^\eta = (\mathcal{S}^\eta, \eta)$ (note that $\bar{A}^\eta \subseteq \{\varphi\} \cup \mathcal{K}$), and*
- *$\mathcal{S}$ is the result of replacing every occurrence of φ in the support $\mathcal{S}^\eta$ by the support $\mathcal{S}^\varphi$.*

3.2 General context dependent preferences

The previous subsection introduced arguments for specificity-based preferences. By allowing rules that specify preferences between defeasible rules in D or between initial information Σ, we enable the derivation of arguments supporting other types of preferences. There are different ways in which we can introduce rules that specify preferences. Here, we choose to extend the definition of a defeasible rule. Alternative choices are special strict rules, or even extending set of atomic propositions used to define the language $\mathcal{L}$ with special atomic propositions that specify preferences between rules in D or between information in Σ. The first alternative is not considered here because it requires a new type of rules that are not a part of the recursive definition of the language $\mathcal{L}$, and the second alternative is not considered because it introduces more expressiveness than needed. So, we allow for additional defeasible rules in D of the form:

$$\alpha \rightsquigarrow (\eta \rightsquigarrow \mu < \varphi \rightsquigarrow \psi) \quad \text{and} \quad \alpha(\mathbf{x}) \rightsquigarrow (\eta(\mathbf{x}) \rightsquigarrow \mu(\mathbf{x}) < \varphi(\mathbf{x}) \rightsquigarrow \psi(\mathbf{x}))$$

where $\{\eta \rightsquigarrow \mu, \varphi \rightsquigarrow \psi\} \subseteq D$ and $\{\eta(\mathbf{x}) \rightsquigarrow \mu(\mathbf{x}), \varphi(\mathbf{x}) \rightsquigarrow \psi(\mathbf{x})\} \subseteq D$ in case of propositional and predicate logic respectively. We also allow for defeasible rules of the form:

$$\alpha \rightsquigarrow (\varphi < \psi)$$

where $\{\varphi, \psi\} \subseteq \Sigma$. These additional defeasible rules allow us construct arguments for preferences that are not based on specificity.

Since we are considering strict preference, opposite preferences must be inconsistent. So given arguments $A = (\mathcal{S}, X < Y)$ and $A' = (\mathcal{S}', Y < Y)$, we can construct a new argument $A'' = (\mathcal{S} \cup \mathcal{S}', \bot)$ for an inconsistency. This argument for an inconsistency is handled in the same way as other arguments for inconsistencies. Another point is the transitive closure of arguments. If desired, a rule for combining the arguments for preferences $X < Y$ and $Y < Z$, can be added.

3.3 The argumentation system

The derivation of (specificity-based) arguments for preferences requires an adaptation of the argumentation system introduced in Section 2. Since preferences are used in resolving derived inconsistencies, Definition 3 must be adapted. A problem that we need to address is that an argument for an inconsistency can be derived before deriving an argument for a relevant preference that can be used to resolve the inconsistency. For this reason we propose to resolve the inconsistency by *explicitly assuming* the absence of preferences between relevant defeasible rule or relevant pieces of information. If an argument for a preference is derived, it will attack the assumption of its absence. Therefore, we propose the following adaptation of Definition 3.

Definition 6 (Definition 3 revised). *Let $A = (\mathcal{S}, \bot)$ be an argument for an inconsistency. Moreover, let $< \subseteq (\Sigma \times \Sigma) \cup (D \times D)$ be a strict partial order over the information Σ and over the defeasible rules D. Finally, let $A' = (\mathcal{S}', \mathbf{not}(\varphi \rightsquigarrow \psi))$ and $A' = (\mathcal{S}', \mathbf{not}(\sigma))$ denote the arguments for an undercutting attack of a defeasible rule in $\overline{D}$ and a proposition in Σ respectively.*

– *If $\tilde{A} \neq \varnothing$, **defeat the weakest last rule**. Let $M = \min_<(\vec{A}^\bot)$ be the set of least preferred last rules for the inconsistency given the fixed preference relation $<$. For every $\varphi \rightsquigarrow \psi \in M$ with $S = (\mathcal{S}'', \varphi \rightsquigarrow \psi) \in \mathcal{S}$,*

$$A' = \big(\mathcal{S} \backslash S \cup \{\boldsymbol{assume}(\eta \rightsquigarrow \mu \not< \varphi \rightsquigarrow \psi) \mid \eta \rightsquigarrow \mu \in M \backslash \varphi \rightsquigarrow \psi\}, \mathbf{not}(\varphi \rightsquigarrow \psi)\big)$$

  is an undercutting argument of $\varphi \rightsquigarrow \psi \in \overline{D}$.
– *If $\tilde{A} = \varnothing$, **defeat the weakest premise**. For every $\sigma \in min_<(\bar{A})$,*

$$A' = \big(\mathcal{S} \backslash \sigma \cup \{\boldsymbol{assume}(\delta \not< \sigma) \mid \delta \in min_<(\bar{A}) \backslash \sigma\}, \mathbf{not}(\sigma)\big)$$

  is an undercutting argument of $\sigma \in \Sigma$.

4 Evaluation

We will evaluate the proposed approach for handling context dependent (specificity) preferences using several problematic examples that have described in the literature. The examples are often used to falsify preceding approaches. We start with the example of Dung and Son [6] and its extension, which was described in the Introduction of this paper. After the examples, we will briefly address the postulates of Dung et al. [7] and the relation with the work of Prakken and Sartor [14].

The extended example of Dung and Son We investigate Dung and Son's example of a context dependent specificity preference [6]. We use the defeasible rules D:

$$\begin{array}{l} student \rightsquigarrow \neg married \\ student \rightsquigarrow young\ adult \\ adult \rightsquigarrow married \end{array}$$

Given the information $\Sigma = \{student, young\ adult \to adult\}$, we can construct the following relevant arguments:

$$
\begin{array}{l}
A_1 = [student \vdash student \rightsquigarrow \neg married \vdash \neg married] \\
A_2 = [student \vdash student \rightsquigarrow young\ adult \vdash adult \rightsquigarrow married \vdash married] \\
A_3 = \left[\begin{array}{r|} student \vdash student \rightsquigarrow \neg married \\ student \vdash student \rightsquigarrow young\ adult \vdash adult \rightsquigarrow married \end{array} \vdash \bot \right] \\
A_4 = \left[\begin{array}{r|} student \vdash student \rightsquigarrow \neg married \\ \mathbf{assume}(student \rightsquigarrow \neg married \not< adult \rightsquigarrow married) \end{array} \circ\, \mathbf{not}(adult \rightsquigarrow married) \right] \\
A_5 = \left[\begin{array}{r|} student \vdash student \rightsquigarrow young\ adult \vdash adult \rightsquigarrow married \\ \mathbf{assume}(adult \rightsquigarrow married \not< student \rightsquigarrow \neg married) \end{array} \circ\, \mathbf{not}(student \rightsquigarrow \neg married) \right] \\
A_6 = [\underline{student} \vdash student \rightsquigarrow young\ adult \vdash adult] \\
A_7 = [student \vdash student \rightsquigarrow young\ adult \mathrel{|\!\circ} adult \rightsquigarrow married < student \rightsquigarrow \neg married]
\end{array}
$$

Note that it is a coincident that the hypothesis $\underline{student}$ in arguments A_6 is the same as the information in Σ. To indicate that $\underline{student}$ is a hypothesis that is used to derive an argument for the specificity preference, we underline *student*. Also note that A_6 is an auxiliary argument that is only used to derive A_7 and has no role in the final set of arguments.

Argument A_4 attacks arguments A_2, A_3 and A_5 ($A_4 \longrightarrow A_2, \ldots$), argument A_5 attacks arguments A_1, A_3 and A_4, and argument A_7 attacks argument A_5. Without the specificity argument A_7, both the stable and the preferred semantics give us two argument extensions: $\{A_1, A_4\}$ and $\{A_2, A_5\}$. The two extension indicate that *we do not know whether the student is married.* After deriving the specificity argument A_7, we have only one argument extension: $\{A_1, A_4, A_7\}$. The latter extension indicates that *the student is not married.*

If we also know that the student is an adult but not a young adult: $\Sigma' = \{student, adult, \neg young\ adult, t, young\ adult \to adult\}$, we can derive the following additional arguments:

$$
\begin{array}{l}
A_8 = [adult \vdash adult \rightsquigarrow married \vdash married] \\
A_9 = \left[\begin{array}{r|} student \vdash student \rightsquigarrow \neg married \\ adult \vdash adult \rightsquigarrow married \end{array} \vdash \bot \right] \\
A_{10} = \left[\begin{array}{r|} student \vdash student \rightsquigarrow \neg married \\ \mathbf{assume}(student \rightsquigarrow \neg married \not< adult \rightsquigarrow married) \end{array} \circ\, \mathbf{not}(adult \rightsquigarrow married) \right] \\
A_{11} = \left[\begin{array}{r|} adult \vdash adult \rightsquigarrow married \\ \mathbf{assume}(adult \rightsquigarrow married \not< student \rightsquigarrow \neg married) \end{array} \circ\, \mathbf{not}(student \rightsquigarrow \neg married) \right] \\
A_{12} = \left[\begin{array}{r|} \neg young\ adult \\ student \vdash student \rightsquigarrow young\ adult \end{array} \vdash \bot \right] \\
A_{13} = [\neg young\ adult \mathrel{|\!\circ} \mathbf{not}(student \rightsquigarrow young\ adult)]
\end{array}
$$

These additional arguments extend the attack relation. Arguments A_4 and A_{10} both attack arguments A_2, A_3, A_5, A_8, A_9 and A_{11}, argument A_5 and A_{11} both attack arguments A_1, A_3, A_4, A_9, A_{10}, argument A_7 attacks arguments A_5 and A_{11}, and argument A_{13} attacks arguments A_2, A_3, A_5, A_7, and A_{12}. Given these attack relations both the stable and the preferred semantics give us two argument extensions: $\{A_1, A_4, A_{10}, A_{13}\}$ and $\{A_2, A_5, A_8, A_{11}, A_{13}\}$. The two extensions indicate that *we do not know whether the student is married.*

Extending the information Σ with *Utopia student* and the defeasible rules D with $Utopia\ student \rightsquigarrow \mathbf{not}(student \rightsquigarrow \neg married)$, we can derive the argument:

$$A_{14} = [\mathit{Utopia\ student} \vdash \mathit{Utopia\ student} \rightsquigarrow \mathbf{not}(\mathit{student} \rightsquigarrow \neg \mathit{married}) \\ \vdash \mathbf{not}(\mathit{student} \rightsquigarrow \neg \mathit{married})]$$

This argument attacks arguments A_1, A_3, A_4. As a result we have only one argument extension: $\{A_2, A_5, A_{14}\}$. This extension indicates that *the student is married*. Note that we do not consider arguments $A_8, \ldots, A_{13}$ here because we do not use the information that the student is an adult but not a young adult.

The example of Modgil Modgil formulates his motivating example in terms of natural language sentences with which he associates argument and attack relations [11]. We first need to reformulate his example in the language of a logic. We choose propositional logic to keep things simple.

- Today will be dry in London since the BBC forecast sunshine: $bs \rightsquigarrow d$.
- Today will be wet in London since CNN forecast rain: $cr \rightsquigarrow w$.
- additional information: bs, cr and $\neg(d \wedge w)$.
- The BBC is more trustworthy than CNN: $bt \rightsquigarrow (cr \rightsquigarrow w < bs \rightsquigarrow d)$
- Statistically CNN is a more accurate forecaster than the BBC: $ca \rightsquigarrow (bs \rightsquigarrow d < cr \rightsquigarrow w)$
- additional information: bt and ca.
- Basing a comparison on statistics is more rigorous and rational than basing a comparison on your instincts about their relative trustworthiness: $\top \rightsquigarrow (bt \rightsquigarrow (cr \rightsquigarrow w < bs \rightsquigarrow d) < ca \rightsquigarrow (bs \rightsquigarrow d < cr \rightsquigarrow w))$.

Using the formulation of the problem in propositional logic, we can derive the following arguments:

$$A_1 = [bs \vdash bs \rightsquigarrow d \vdash d]$$

$$A_2 = [cr \vdash cr \rightsquigarrow w \vdash w]$$

$$A_3 = \left[\left. \begin{array}{r} bs \vdash bs \rightsquigarrow d \\ cr \vdash cr \rightsquigarrow w \\ \neg(d \wedge w) \end{array} \right| \vdash \bot \right]$$

$$A_4 = \left[\left. \begin{array}{r} bs \vdash bs \rightsquigarrow d \\ \neg(d \wedge w) \\ \mathbf{assume}(bs \rightsquigarrow d \nless cr \rightsquigarrow w) \end{array} \right| \circ\, \mathbf{not}(cr \rightsquigarrow w) \right]$$

$$A_5 = \left[\left. \begin{array}{r} cr \vdash cr \rightsquigarrow w \\ \neg(d \wedge w) \\ \mathbf{assume}(cr \rightsquigarrow w \nless bs \rightsquigarrow d) \end{array} \right| \circ\, \mathbf{not}(bs \rightsquigarrow d) \right]$$

$$A_6 = [bt \vdash bt \rightsquigarrow (cr \rightsquigarrow w < bs \rightsquigarrow d) \vdash cr \rightsquigarrow w < bs \rightsquigarrow d]$$

$$A_7 = [ca \vdash ca \rightsquigarrow (bs \rightsquigarrow d < cr \rightsquigarrow w) \vdash bs \rightsquigarrow d < cr \rightsquigarrow w]$$

$$A_8 = \left[\left. \begin{array}{r} bt \vdash bt \rightsquigarrow (cr \rightsquigarrow w < bs \rightsquigarrow d) \\ ca \vdash ca \rightsquigarrow (bs \rightsquigarrow d < cr \rightsquigarrow w) \end{array} \right| \vdash \bot \right]$$

$$A_9 = \left[\left. \begin{array}{r} bt \vdash bt \rightsquigarrow (cr \rightsquigarrow w < bs \rightsquigarrow d) \\ \mathbf{assume}(bt \rightsquigarrow (cr \rightsquigarrow w < bs \rightsquigarrow d) \nless \\ ca \rightsquigarrow (bs \rightsquigarrow d < cr \rightsquigarrow w)) \end{array} \right| \circ\, \mathbf{not}(ca \rightsquigarrow (bs \rightsquigarrow d < cr \rightsquigarrow w)) \right]$$

$$A_{10} = \left[\left. \begin{array}{r} ca \vdash ca \rightsquigarrow (bs \rightsquigarrow d < cr \rightsquigarrow w) \\ \mathbf{assume}(ca \rightsquigarrow (bs \rightsquigarrow d < cr \rightsquigarrow w) \nless \\ bt \rightsquigarrow (cr \rightsquigarrow w < bs \rightsquigarrow d)) \end{array} \right| \circ\, \mathbf{not}(bt \rightsquigarrow (cr \rightsquigarrow w < bs \rightsquigarrow d)) \right]$$

$$A_{11} = [\vdash \top \rightsquigarrow (bt \rightsquigarrow (cr \rightsquigarrow w < bs \rightsquigarrow d) < ca \rightsquigarrow (bs \rightsquigarrow d < cr \rightsquigarrow w)) \vdash \\ bt \rightsquigarrow (cr \rightsquigarrow w < bs \rightsquigarrow d) < ca \rightsquigarrow (bs \rightsquigarrow d < cr \rightsquigarrow w)]$$

Argument A_4 attacks arguments A_2 and A_3, argument A_5 attacks arguments A_1 and A_3, argument A_6 attacks argument A_5, argument A_7 attacks argument A_4, argument A_9 attacks arguments A_7 and A_8, argument A_{10} attacks arguments A_6 and A_8, and argument A_{11} attacks argument A_9. Given these attack relations both the stable and the preferred semantics give us one argument extension: $\{A_2, A_5, A_7, A_{10}, A_{11}\}$. Hence, *today will be wet in London since CNN forecast rain.*

The example of Amgoud and Vesic Amgoud and Vesic [1] argue that Modgil's approach [11] can result in extensions of which the arguments support inconsistent conclusions. These inconsistencies are not visible at the level of the argumentation framework because arguments are viewed as atoms. The solution of Amgoud and Vesic is to reverse the attack relation. This solution can be invalid if the attack relation is a result of an undercutting attack.

The approach described in this paper correctly handles the motivating example of Amgoud and Vesic [1]. Since this example is formulated in terms of natural language sentences with which argument and attack relations are associated, first need to reformulate the example in the language of a logic.

- This violin is expensive since it was made by Stradivari: $s \leadsto e$.
- additional information: s
- The violin was not made by Stradivari: $\neg s$.
- since the first statement is from an expert while the second is from a child, we have the preference relation: $\top \leadsto \neg s < s$.

Using the formulation of the problem in propositional logic, we can derive the following arguments:

$$
\begin{aligned}
A_1 &= [s \vdash s] \\
A_2 &= [s \vdash s \leadsto e \vdash e] \\
A_3 &= [\neg s \vdash \neg s] \\
A_4 &= \left[\begin{array}{c|} s \\ \neg s \end{array} \vdash \bot \right] \\
A_5 &= \left[\begin{array}{r|} s \\ \mathbf{assume}(s \not< \neg s) \end{array} \circ \mathbf{not}(\neg s) \right] \\
A_6 &= \left[\begin{array}{r|} \neg s \\ \mathbf{assume}(\neg s \not< s) \end{array} \circ \mathbf{not}(s) \right] \\
A_7 &= [\,\vdash \top \leadsto \neg s < s \vdash \neg s < s]
\end{aligned}
$$

Argument A_5 attacks arguments A_3, A_4 and A_6, argument A_6 attacks arguments A_1, A_2, A_4 and A_5, and argument A_7 attacks argument A_6. Given these attack relations both the stable and the preferred semantics give us one argument extension: $\{A_1, A_2, A_5, A_7\}$. Hence, *this violin is expensive since it was made by Stradivari.*

The example of Dung, Thang and Son Dung et al. [7] use the following example formulated in predicate logic extended with defeasible rules.

> Sherlock Holmes is investigating a case involving three persons P_1, P_2 and S together with the dead body of a big man. Furthermore, S is a small child who cannot kill a big man and P_1 is a beneficiary from the dead of the big man.

Dung et al. [7] provide the following information and defeasible rules about the case:

1. The knowledge that one of the persons is the murderer is represented by three strict rules:
   $Inno(P_1) \wedge Inno(S) \rightarrow \neg Inno(P_2)$
   $Inno(P_2) \wedge Inno(S) \rightarrow \neg Inno(P_1)$
   $Inno(P_1) \wedge Inno(P_2) \rightarrow \neg Inno(S)$
2. The legal principle that people are considered innocent until proven otherwise could be represented by three defeasible rules:
   $\top \rightsquigarrow Inno(P_1)$
   $\top \rightsquigarrow Inno(P_2)$
   $\top \rightsquigarrow Inno(S)$
3. A "rule-of-thumb" for the investigation is to find out whether the possible suspects have any motives and to focus the investigation on the one with strong motive to commit the crime. Such "rule-of-thumb" can be represented by two conditional preferences:
   $Has\_Motive(P_1) \wedge \neg Has\_Motive(P_2) \rightsquigarrow (\top \rightsquigarrow Inno(P_1) < \top \rightsquigarrow Inno(P_2))$
   $Has\_Motive(P_2) \wedge \neg Has\_Motive(P_1) \rightsquigarrow (\top \rightsquigarrow Inno(P_2) < \top \rightsquigarrow Inno(P_1))$
   The rules state that if P_i has a motive and P_j $(i \neq j)$ does not have a motive then the default that P_j is innocent is more preferred than the default that P_i is innocent.
4. A good reason for having a motive to kill is to be a beneficiary from the dead of the deceased:
   $Beneficiary(P_1) \rightarrow Has\_Motive(P_1)$
   $Beneficiary(P_2) \rightarrow Has\_Motive(P_2)$
5. Peoples are normally assumed not to have motives to kill:
   $\top \rightsquigarrow \neg Has\_Motive(P_1)$
   $\top \rightsquigarrow \neg Has\_Motive(P_2)$
6. The information that S is a small child and P_1 is a beneficiary from the dead of the big man is represented by the information:
   $Inno(S), Beneficiary(P_1)$

After deriving all relevant arguments and attack relations for the above example[3], we can identify one extension for both the stable and the preferred semantics, which supports the conclusion that person P_2 and child S innocent and person P_1 is not innocent.

The postulates of Dung, Thang and Son Beside the above example, Dung et al. [7] also introduce postulates. These postulates are not relevant here because they

[3] We do not have the space to list all relevant arguments and he attack relations that they imply here.

specifically address arguments for preferences attacking attack relations. This is not the approach used here, and therefore, the postulates are not relevant in the here proposed approach.

The relation with the approach of Prakken and Sartor The introduction of assumptions that no other last rule has a lower preference (Definition 6), does not change the original argumentation system. Without arguments attacking these assumptions, the same conclusions will be supported given an argumentation semantics. Arguments for preference between defeasible rules can affect the supported conclusions by attacking the added assumption.

Comparing the here proposed approach with the one proposed by Prakken and Sartor [14], it is not difficult to verify that the application of the characteristic function has the same result in both approaches in case of the grounded semantics. Prakken and Sartor use 'justified arguments' for a preferences to determine a new set of acceptable arguments. In the here proposed approach, these 'justified arguments' for preferences successfully attack one of the undercutting arguments that are generated to resolve rebutting conflicts. If an argument for a conclusion is successfully attacked, the rebutting conclusion will be acceptable. So, the here proposed approach can be viewed as a generalization of the approach proposed by Prakken and Sartor [14].

5 Conclusion

We have investigated a new way of handling context dependent (specificity) preferences. Undercutting argument in which we explicitly assume the absence of preferences between defeasible rules or between premises, handle the arguments for inconsistencies in the proposed approach. These undercutting arguments can be attacked by arguments for preferences. The approach is intuitively simple and can handle examples that have been used to motivate alternative approaches. We therefore conclude that the propose approach is able to adequately handle context dependent (specificity) preferences. Moreover, because the approach in intuitively more simple than alternative approaches that have been proposed in the literature, the proposed approaches is to be preferred over the alternatives. Finally, we conclude that it is important to consider the defeasible elements that make up an argument instead of viewing the argument as an atom.

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